Stats 1 - January 2010

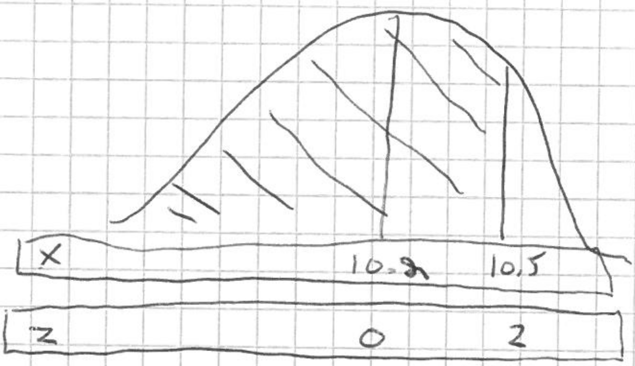
① $X \sim N(10.2, 0.15^2)$

a) i) $P(X < 10.5)$

$= P\left(Z < \frac{10.5 - 10.2}{0.15}\right)$

$= P(Z < 2)$

$= 0.97725$



a) ii) $P(10.0 < X < 10.5)$

$= P\left(\frac{10 - 10.2}{0.15} < Z < \frac{10.5 - 10.2}{0.15}\right)$

$= P(-1.33 < Z < 2)$

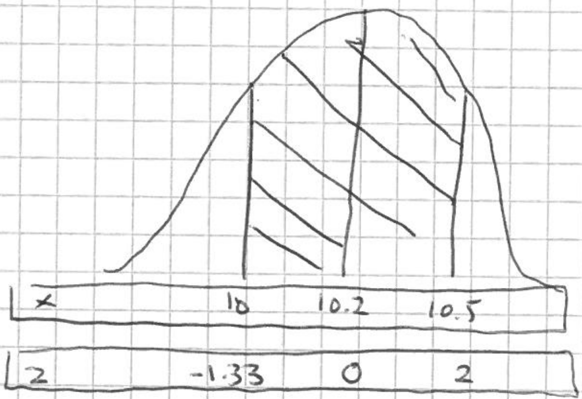
$= P(Z < 2) - P(Z < -1.33)$

$= 0.97725$

$= P(Z > 1.33)$

$= 1 - P(Z < 1.33)$

$= 1 - 0.90824 = 0.09176$



$= 0.97725 - 0.09176 = 0.88549$

b) $P(X > 10) = 0.90824$ from a ii)

$P(6 \text{ rolls}) = (0.90824)^6 = 0.561311\ldots$

② a) a 14 15 18 20 25 25 $\boxed{26}$ 27 29 32 34 37 37 b

Median $= \frac{15+1}{2} = 8\text{th} = 26$

LQ $= \frac{15+1}{4} = 4\text{th} = 18$

UQ $= \frac{3(15+1)}{4} = 12\text{th} = 34$

IQR $= 34 - 18 = 16$

b) Mode – No unique value, ∴ 2 modes of 25 & 3

Standard Deviation – we don't know the values of a and

c) From calculator: $\Sigma x = 390$ $\quad \Sigma x^2 = 11472$

Mean $(\bar{x}) = 26$

Sample SD $(s) = 9.7541\ldots$

(3) a) From calculator: $\Sigma x = 1351$

$a = 2501.091046\ldots$ (intercept)

$b = 7.05134\ldots$ (gradient)

$\Rightarrow \quad y = 2501.1 + 7.051x$

b) $x = 200 \quad \rightarrow \quad y = 2501.1 + 7.051(200)$

$= 3911$ litres

c) Residuals are only approximately 10% of the y values, ∴ not too bad an estimate (fairly reliable).

(4) a) i) $P(R_W, S_W, T_W) = 0.65 \times 0.4 \times 0.25$
$= 0.065$

ii) $P(R_B, S_{B'}, T_{B'}) = 0.25 \times 0.85 \times 0.8$
$= 0.17$

iii)
$P(R_C, S_C, T_C) = 0.1 \times 0.45 \times 0.55 = 0.02475$

$P(R_C, S_C, T_{C'}) = 0.1 \times 0.45 \times 0.45 = 0.02025$

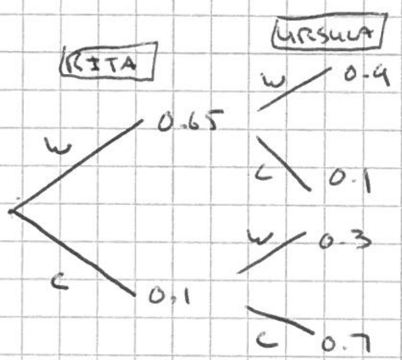
$P(R_C, S_{C'}, T_C) = 0.1 \times 0.55 \times 0.55 = 0.03025$

$P(R_{C'}, S_C, T_C) = 0.9 \times 0.45 \times 0.55 = 0.22275$

+

0.298

b)



i) $P(\text{same}) = P(W,W) + P(C,C)$

$= 0.65 \times 0.9 + 0.1 \times 0.7$

$= 0.655$

$P(\text{different}) = 1 - 0.655$

$= 0.345$

(5) a) i) From calculator:

$\bar{x} = 1010 \quad \sigma = 10.5 \quad n = 12$

98% multiplier for Z (2 tailed) = 2.3263

98% CI for $\mu = \bar{x} \pm Z \times \frac{\sigma}{\sqrt{n}}$

$= 1010 \pm 2.3263 \times \frac{10.5}{\sqrt{12}}$

$= 1010 \pm 7.05122...$

$= (1003, 1017)$ (4 s.f.)

ii) The population is normally distributed

iii) About 30

b) $\boxed{CI}$ 1kg is outside the confidence interval. It is lower than the lower bound of 1003

$\boxed{SAMPLE}$ 3/12 or 25% of weights below 1kg

∴ Likely to be invalid labelling.

c) 2% (100 − 98)

(6) Let X = number of nights out

a) $X \sim B(14, 0.35)$

i) $P(X \leq 7) = 0.9247$ (tables)

ii) $P(X \geq 11) = 1 - P(X \leq 10)$

$= 1 - 0.9989 = 0.0011$

iii) $P(5 < X < 10)$

CAN BE: 6, 7, .. 9

$= P(X \leq 9) - P(X \leq 5)$

$= 0.9940 - 0.6405 = 0.3535$

b) 3 weeks = 21 days → $X \sim B(21, 0.35)$

$P(X = 4) = {}^{21}C_4 \times 0.35^4 \times 0.65^{17}$

$= 0.05927...$

c) $S \sim B(7, 5/7)$

i) MEAN $= np = 7 \times 5/7 = 5$

VARIANCE $= np(1-p) = 7 \times 5/7 \times 2/7 = 10/7$ or $1.43\ldots$

ii) Mean is the same and variance is similar.

$\therefore$ Barry's claim seems valid.

(7) a) From calculator: $\Sigma x = 636$

$r = -0.03546\ldots$

b) Virtually no linear correlation between purchase price and auction price.

c) i) See Mark Scheme

ii) Positive linear correlation between most of the data

BUT

Two outliers: J and L

d) i) $r = \dfrac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}} = \dfrac{4268.8}{\sqrt{4854.4 \times 4216.1}}$

$= 0.943588\ldots$

ii) Strong, positive, linear correlation between purchase price & auction price.